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Gamma Ray Burst Source Statistics in the Presence of Stochastic Errors

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Abstract

Selection effects, connected with stochastic errors in source flux and threshold value determination are analyzed. Normal and normal logarithmic distributions of stochastic deviations are considered. These two kind of distributions produce different effects on the source statistics. Applications to Gamma Ray Burst statistics are discussed. A physical test for checking a close neutron star model of GRBs is suggested.

1 Introduction

Statistical investigation of samples of sources is a powerful method for analyzing their location, origin and evolution. Most important results were obtained for distant radio sources (Longair, 1966). In combination with redshift measurements these data permit also to get estimates for cosmological parameters (Zeldovich and Novikov, 1975).

The BATSE (Meegan et al,1992; Kouveliotou, 1994; Fishman and Meegan, 1995) curve $[\log N - \log(C/C_{min})]$ gives very important information for making constraints on GRB models and understanding their nature. Nevertheless, it suffers from different selection effects, so it seems premature to use it for a critical choice of GRB models. This curve differs from the straight line with a slope 3/2, and the observed isotropy of GRB distribution on the sky is consistent with the following models:

1) Nearby neutron stars from the disc population.

2) Galactic halo neutron stars.

3) Cosmological model with bursts coming from sources with redshifts $z \simeq 1 \div 2$.

The last two models also explain deviations from the 3/2 line. The third one has extreme demands (neutron star collision): huge energy release in soft gamma region with no counterpart in optics or radio. While some theoretical models

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based on fireball expansion (Mészáros & Rees,1993) seems available to reproduce the main properties of GRB, it is questionable to get such a fireball in neutron star collision (Isern et.al.,1995). There is also a problem of extended hard gamma - emission, accompanying the main burst (Hurley et.al.,1994). The halo model is posing restrictions (Hakkila et al,1995; Bulik and Lamb, 1995) to the properties of the neutron star kinematics (speed at the origin $\sim 10^3$ km/s), GRB fluence (narrow strip around 10^{42} ergs) and location (at the outer edge of a sphere with $R \simeq 350$ kpc).

Here we investigate the influence of stochastic errors on the shape of the $[\log N - \log(C/C_{min})]$ curve. Such type of errors have been first taken into account by Eddington (1913,1940), who considered small stochastic errors in determination of the observed magnitude of stars. Fluence or peak flux of GRBs are determined with much larger errors, related to the uncertainty in the burst angular direction, its spectral and temporal variability, background fluctuations.

2 Source statistics in presence of stochastic errors

Let χ be a number density of events, registered by an observer with a flux C. If ν is a constant burst frequency per unit volume, L is a constant peak luminosity, C_x is a variable threshold flux, $g(C_x)$ is a threshold distribution function, then, following Schmidt et al (1988), Band (1992), Petrosian (1993) we may write

$$\chi(C) \,=\, 4\pi\nu \int \delta(C - \frac{L}{4\pi r^2})\, \theta(C - C_x)\, g(C_x) r^2\, dr\, dC_x \tag{1}$$

a) Logarithmic normal distribution. Consider first a classical case where threshold effects are neglected. Actually, Eddington (1913,1940) considered a case with no threshold, when all sources could be registered. For that case we have instead of (1)

$$\chi(C) \,=\, 4\pi\nu \int \delta(C - \frac{L}{4\pi r^2})\, r^2\, dr \tag{2}$$

which gives

$$\chi(C) \,=\, \left(\frac{L}{4\pi}\right)^{3/2} \frac{2\pi\nu}{C^{5/2}} \tag{3}$$

Following Eddington (1913), take errors, according to a normal Gaussian distribution over $\log C$. The observed flux C could be produced by the source, whose real flux is C' with the probability

$$\frac{1}{\Delta\sqrt{\pi}} \exp\left[-\frac{(\log C' - \log C)^2}{\Delta^2}\right] d(logC') \tag{4}$$

The observed number density of events $\tilde{\chi}(C)$ is connected to the real distribution $\chi(C')$ by the relation

$$\tilde{\chi}(C) \; = \frac{1}{\Delta\sqrt{\pi}} \int_{-\infty}^{\infty} \tag{5}$$

$$\times \chi(C') \; \exp\left[-\frac{(\log C' - \log C)^2}{\Delta^2}\right] d(\log C')$$

Using (3) in (5) we get after integration

$$\tilde{\chi}(C) \; = \chi(C) \, e^{\frac{25}{16}\Delta^2} \tag{6}$$

So, Gaussian logarithmic statistical errors in absence of threshold does not change the slope of the number density curve, increasing it by a constant coefficient (Eddington, 1913)

b) Normal logarithmic distribution with variable threshold. For a variable threshold with $g(C_x) \neq \delta(C_0)$ we need to consider a distribution (Schmidt et.al.,1988) over $\xi = \frac{C}{C_x}$

$$\chi(\xi) = \; 4\pi\nu \int \delta(\xi C_x - \frac{L}{4\pi r^2}) \tag{7}$$

$$\times \theta(\xi C_x - C_x) \, g(C_x) r^2 \, dr \, dC_x$$

which gives after integration

$$\chi(\xi) \; = \; \left(\frac{L}{4\pi}\right)^{3/2} \frac{2\pi\nu A}{\xi^{5/2}} \theta(\xi - 1) \tag{8}$$

with

$$A = \int_0^{\infty} \frac{g(C_x) \, dC_x}{C_x^{5/2}}$$

Consider now a Gaussian logarithmic distribution of errors like (4) with ξ and ξ' instead of C and C'. While it is impossible to register events under the threshold, we shall use the interval $1 < \xi' < \infty$ for possible real values. Then we have instead of (5)

$$\tilde{\chi}(\xi) \; = \frac{1}{B} \int_0^{\infty} \chi(\xi') \; \exp\left[-\frac{(\log \xi' - \log \xi)^2}{\Delta^2}\right] d(\log \xi') \tag{9}$$

with

$$B = \int_0^{\infty} \exp\left[-\frac{(x - \log \xi)^2}{\Delta^2}\right] dx$$

Using (8) in (9) we get

$$\tilde{\chi}(\xi) = \left(\frac{L}{4\pi}\right)^{3/2} \frac{2\pi\nu A\, I}{B\xi^{5/2}} \tag{10}$$

Here

$$\begin{aligned} B &= \Delta\Big(\int_0^\infty e^{-z^2}\, dz + \int_0^{(\log\xi)/\Delta} e^{-z^2}\, dz\Big) \qquad (11)\\ &= \Delta\frac{\sqrt{\pi}}{2}\big[1 + Erf\left(\frac{\log\xi}{\Delta}\right)\big], \\ I &= \Delta e^{\frac{25}{16}\Delta^2}\Big(\int_{\frac{5}{4}\Delta}^\infty e^{-z^2}\, dz + \int_{-\frac{5}{4}\Delta}^{\frac{\log\xi}{\Delta}-\frac{5}{4}\Delta} e^{-z^2}\, dz\Big) \\ &= \Delta\frac{\sqrt{\pi}}{2} e^{\frac{25}{16}\Delta^2}\big[1 + Erf\left(\frac{\log\xi}{\Delta} - \frac{5}{4}\Delta\right)\big]. \end{aligned}$$

where $Erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-z^2}\, dz$. An observed distribution $\tilde{\Xi}(\xi)$ obtained by integration of (10) over $d\xi$ is written as follows

$$\begin{aligned} \tilde{\Xi}(\xi) &= \int_\xi^\infty \tilde{\chi}(\xi')\, d\xi' \qquad (12)\\ &= \left(\frac{L}{4\pi}\right)^{3/2} 2\pi\nu A\, e^{\frac{25}{16}\Delta^2} \int_\xi^\infty \frac{D(x)}{x^{5/2}} dx \end{aligned}$$

With

$$D(x) = \frac{1 + Erf\left(\frac{\log\xi}{\Delta} - \frac{5}{4}\Delta\right)}{1 + Erf\left(\frac{\log\xi}{\Delta}\right)} \tag{13}$$

The function $\log\tilde{\Xi}$ as a function of $\log\xi$ is represented in fig.1 for $\Delta = 1$. The corresponding curve for $\Delta = 0.1$ is cannot be distinguished from the straight line with slope 3/2 indicated there. Contrary to the case with no threshold (Eddington,1913), where stochastic errors do not change the slope of the curve $[\log N - \log C]$, stochastic errors in presence of threshold may considerably decrease the slope of the curve in the vicinity of the threshold. It happens, because the threshold is acting like a border, which cannot be crossed by the bursts from both sides. So faint bursts appear more like stronger ones, and the spreading of stronger bursts is almost equal in both directions.

c) Normal distribution. While we do not know exactly what law is determining stochastic errors, let us consider also a normal distribution of errors around the value ξ itself. While our eye has a logarithmic response to the signal, some X - ray counters are proportional. May be the true distribution is even more complicated and does not follow a Gaussian law neither in logarithms nor in the values themselves.

Figure 1: The curve $[\log N - \log C(max)/C(thr)]$ in presence of stochastic errors, distributed according to normal logarithmic distribution; 1 - straight line with a slope 3/2, corresponding to $\Delta = 0$; 2 - curve with $\Delta = 1$; $C(max)$ is the peak intensity of the burst; $C(thr)$ is a corresponding threshold value.

Assume that the burst, registered with the ratio $\xi = C/C_x$ may in reality correspond to the ratio ξ' with the probability

$$\sim \exp\left[-\frac{(\xi' - \xi)^2}{\Delta_1^2}\right] d\xi' \tag{14}$$

Then taking into account only the events over the threshold we get instead of (5)

$$\tilde{\chi}(\xi) = \frac{1}{B_1} \int_1^\infty \chi(\xi') \exp\left[-\frac{(\xi' - \xi)^2}{\Delta_1^2}\right] d\xi' \tag{15}$$

with

$$B_1 = \int_1^\infty \exp\left[-\frac{(\xi' - \xi)^2}{\Delta_1^2}\right] d\xi' \tag{16}$$

Taking into account the (3) we get

$$\tilde{\chi}(\xi) \;=\; \left(\frac{L}{4\pi}\right)^{3/2} \frac{2\pi\nu A\, I_1}{\Delta_1^{3/2} B_1}, \tag{17}$$

where the integrals may be expressed as

$$B_1 = \Delta_1 \frac{\sqrt{\pi}}{2} \big[1 + Erf\left(\frac{\xi - 1}{\Delta_1}\right)\big], \tag{18}$$

$$I_1 = \int_{\frac{1}{\Delta_1}}^\infty y^{-5/2} \exp\big[-\left(y - \frac{\xi}{\Delta_1}\right)^2\big]\, dy. \tag{19}$$

For the distribution we are looking for we get

$$\tilde{\Xi}(\xi) = \left(\frac{L}{4\pi}\right)^{3/2} \frac{4\pi\nu A}{\sqrt{\pi}\Delta_1^{5/2}} \int_\xi^\infty D_1(x) dx \tag{20}$$

with

$$D_1(x) = \frac{I_1(x)}{1 + Erf\left(\frac{x-1}{\Delta_1}\right)} \tag{21}$$

For sufficiently large values of ξ, when the function $Erf(x)$ is very close to unity, the expression for $\tilde{\Xi}(\xi)$ in (20) may be written as

$$\tilde{\Xi}(\xi) = \left(\frac{L}{4\pi}\right)^{3/2} \frac{4\pi\nu A}{3\sqrt{\pi}\Delta_1^{3/2}} \left[\int_0^{\frac{\xi-1}{\Delta_1}} \left(\frac{\xi}{\Delta_1} - z\right)^{-3/2} \right. \tag{22}$$

$$\left. \times e^{-z^2} dz + \int_0^\infty \left(\frac{\xi}{\Delta_1} + z\right)^{-3/2} e^{-z^2} dz \right]$$

The plot of $\log \Xi(\xi)$ as a function of $\log \xi$ is represented in fig.2 for $\Delta_1 = 1,\ 10$, together with the straight line with the slope 3/2.

3 Discussion

BATSE consists (Fishman, 1992) of eight detectors, arranged on corners of the Compton Gamma Ray Observatory (CGRO). Burst registration is done by large area detectors (LAD), optimized sensitivity and directional response. The eight panels of LAD are parallel to the eight faces of a regular octahedron. Since a regular octahedron is comprised of four sets of parallel intersecting planes, every detected burst will be viewed by four detectors. LAD are sensitive in the energy range 20-600 keV. The burst is registered, when 5.5σ excess over 17 s background rate is registered at least by two detectors (Fishman et al, 1992). The background in LAD in the burst trigger energy range 60-300 keV varies between approximately 1500 counts/s and 3000 counts/s per detector during most portion of the orbit and above geographic latitudes of about 22° the background increases considerably.

Each event with the ratio ξ of the peak luminosity to a local background is detected with an error due to the following circumstances.

1) Spectral dispersion of GRBs cannot guarantee that a peak value in the BATSE spectral region (BSR) is equal to a real one. It may be several times larger if the region of maximum radiation lay outside BSR.

2) Angular dependence of the detector sensitivity, especially in the region of a steep dependence around 50° (Fishman,1992) imply errors in determination of ξ because of poor angular localization of the source.

3) Different time duration of bursts lead to nonuniformity of the source sample, where a same peak luminosity may be related to bursts with a total flux or average luminosity varying by orders of magnitude. Conversely very different peak luminosities may correspond to bursts with the same total flux. This would imitate stochastic errors of the same order.

4) Nonuniformity of GRB detection conditions, when an event may be registered by 2 or 3 or 4 detectors determines an additional source of dispersion. So, uncertainty in the ξ value represented by the dispersion equal to 10 threshold levels, described above, does not seems to be overestimated.

For $\Delta_1 = 10$, corresponding to $\Delta = 1$ in the case of logarithmic normal distribution of errors, the shape of the curve is changed considerably. The changes become noticeable at $\xi \sim 30$ (see fig.2), which is much larger, than the average dispersion value $\Delta_1 = 10$. In the case of logarithmic dispersion the influence of stochastic errors starts at ξ approximately equal to the value of the average dispersion (see fig.1). Results, represented in fig.1,2 illustrate the large importance of different stochastic errors in the form of the $[\log N - \log(C/C_{min})]$ curve and are not intended to explain directly the corresponding BATSE curve.

We have not taken into account other kinds of threshold influence, leading to

Figure 2: Same as in Fig.1 for normal distribution; 1 - straight line with slope 3/2, corresponding to $\Delta_1 = 0$; 2 - curve with $\Delta_1 = 1$; 3 - curve with $\Delta_1 = 10$.

additional deviations from the 3/2 slope (Hartman and The, 1993; Lingenfelter, 1995). Combination of these effects must be taken into account in analyzing the $[\log N - \log(C/C_{min})]$ curve for the BATSE data sample.

Let us note, that stochastic errors from the photon noise, to which Eddington (1913) applied his calculations, are much smaller than possible errors in BATSE data caused by above mentioned reasons. Even in statistical analysis of G- stars, situated uniformly inside the Galactic disc, the value of $< V/V_{max} >$ which must be equal to 0.5 for uniform unbiased sample, falls down considerably around two threshold flux, which is connected most probably with a loss of faint sources (Harrison et al, 1995).

In view of this situation it seems preliminary to wipe away a close Galactic origin for GRBs and all variety of models (see e.g. Ho et al, 1992) should remain under discussion.

4 Testing the physical origin of GRBs

If GRBs are connected with starquakes on nearby neutron stars (Bisnovatyi-Kogan et al, 1975) it is worth (Bisnovatyi-Kogan, 1993) to monitor close young pulsars (Geminga) for catching the moment of the quake and comparing it with GRB search data. If the hard tail of GRBs (Hurley et al, 1994) is connected with the excitation of submsec proper oscillations of the neutron star after starquake, and hard gamma ray emission in GRBs is produced by the same mechanism as in radiopulsars (Bisnovatyi-Kogan, 1995), one implicit test for GRB origin may be suggested. If starquakes in radiopulsars lead to the excitation of such oscillations, they can lead to the appearance of resonant modes in a frequency spectrum of radioemission.

The electrical field, generated on the surface of an oscillating neutron star, is of the order of (Muslimov and Tsygan, 1986)

$$E_{osc} \simeq \frac{v}{c} B \approx \frac{\delta R}{R} \frac{v_{ff}}{c} B \approx \frac{\delta R \Omega_{osc}}{c} B \tag{23}$$

When the amplitude of oscillations is large enough

$$\frac{\delta R}{R} \geq \frac{\Omega_{rot}}{\Omega_{osc}} \simeq \frac{10^{-4}}{P_{rot}}, \tag{24}$$

this oscillating field could modulate a pair cascade birth, leading to the appearance of a coherent high frequency mode in the frequency spectrum of radioemission.

In old nearby neutron stars (silent ones) we may expect slower rotation and lower magnetic field than in radiopulsars. These neutron stars could become pulsars (in hard gamma as well as in radio) only temporally, for about a few hours after the quake, and produce a GRB, if the electrical field, induced by oscillations is higher than the threshold field for pair cascade generation.

The best object for such testing is the Vela radiopulsar, where strong glitches are observed almost every year (MacCulloch et.al.,1987). It is necessary to be able to make a frequency analysis of the radio data very soon after the quake for checking the existence of resonance frequencies with periods less then one millisecond. If the model of close Galactic GRB with its logical consequence listed above is true, we may expect to see high frequency resonance oscillations only during a limited period of time of the order of 90 minutes after the visible glitch. Radio observations of Vela soon after glitch with high time resolution and accurate frequency analysis could be more informative than gamma ray observations (Hartmann et al, 1992).

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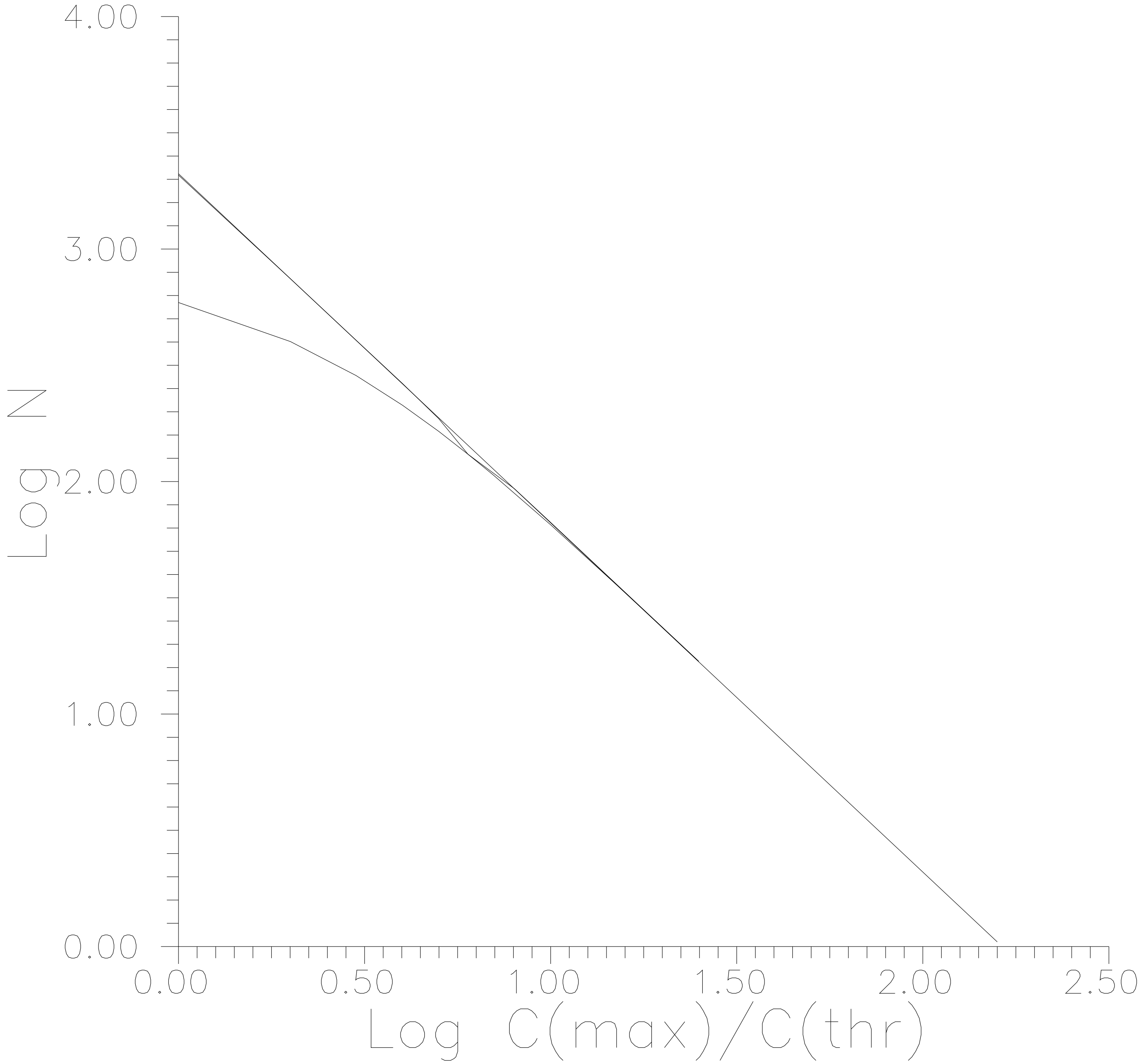

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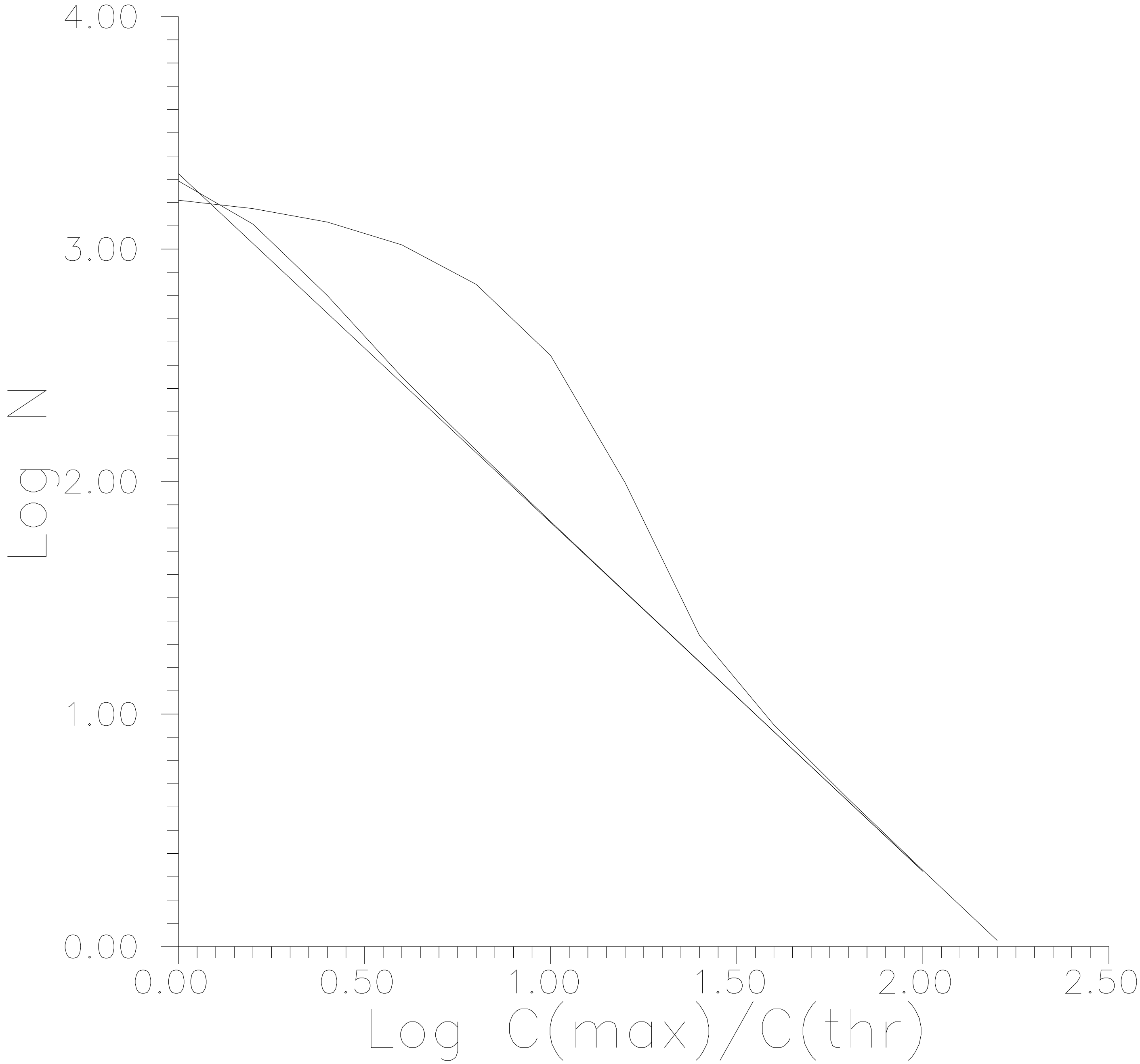
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